THE CORE VARIETY AND OPEN QUESTIONS IN THE MULTIVARIABLE MOMENT PROBLEM

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Abstract. Let $\beta \equiv \beta^{(m)} = \{\beta_i\}_{i \in \mathbb{Z}_+^n, |i| \le m}$, $\beta_0 > 0$, denote a real n-dimensional multisequence of degree m. The *Truncated K-Moment Problem* for β (TMP) concerns the existence of a positive Borel measure μ, supported in K, such that $\beta_i = \int_{\mathbb{R}^n} x^i d\mu$ ($i \in \mathbb{Z}_+^n$, $|i| \le m$). (Here, for $x \equiv (x_1, \ldots, x_n) \in \mathbb{R}^n$ and $i \equiv (i_1, \ldots, i_n) \in \mathbb{Z}_+^n$, we set $|i| = i_1 + \cdots + i_n$ and $x^i = x_1^{i_1} \cdots x_n^{i_n}$.) A measure μ as above is a *K-representing measure* for β. An important result of J. Stochel [Glasg. Math. J. 43 (2001), 335-341] shows that solving the truncated K-moment problem also solves the corresponding *full* K-moment problem for $\beta^{(\infty)}$. For $K = \mathbb{R}^n$ we discuss three equivalent "solutions" to TMP, based on: 1) flat extensions of moment matrices, 2) positive extensions of Riesz functionals, and 3) the *core variety* of a multisequence (introduced by the author in [JMAA 456 (2017), 946-969]). In work with G. Blekherman [J. Operator Theory 84 (2020), 185-209], we proved that β has a representing measure if and only if the core variety is nonempty, in which case the core variety is the union of supports of all finitely atomic representing measures. We discuss examples and open questions related to applying of any of the above solutions to TMP in special cases or in numerical examples.

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