Kernels of Toeplitz operators and de Branges-Rovnyak spaces

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Let H^2 denote the standard Hardy space on the unit disk $\mathbb{D}$ and let $\mathbb{T} = \partial\mathbb{D}$. For $\varphi \in L^\infty(\mathbb{T})$ the Toeplitz operator T_φ on H^2 is given by $T_\varphi f = P_+(\varphi f)$, where P_+ is the orthogonal projection of $L^2(\mathbb{T})$ onto H^2. It is a consequence of Hitt's Theorem that $\ker T_\varphi = fK_I$, where $K_I = H^2 \ominus IH^2$ is the model space corresponding to the inner function I such that $I(0) = 0$ and f is an outer function of unit H^2 norm that acts as an isometric multiplier from K_I onto fK_I.

The sufficient and necessary condition for the space fK_I to be the kernel of a Toeplitz operator was given by E. Hayashi (1990). In 1994 D. Sarason gave another proof of this condition based on de Branges-Rovnyak spaces theory. If $M = fK_I$ is a kernel of a Toeplitz operator, then we also have $M = \ker T_{\frac{\bar{I}\bar{f}}{f}}$. In the talk we consider the case when $fK_I \subsetneq \ker T_{\frac{\bar{I}\bar{f}}{f}}$ and describe the space $\ker T_{\frac{\bar{I}\bar{f}}{f}} \ominus fK_I$ in the case when this space is finite dimensional. We use Sarason's approach and the structure of de Branges-Rovnyak spaces generated by nonextreme functions.

The talk is mainly based on the papers:

- Nowak, M. T., Sobolewski, P., Sołtysiak, A., Wołoszkiewicz-Cyll, M., *On kernels of Toeplitz operators*, Anal. Math. Phys. 10 (2020), no. 4;
- Nowak, M. T., Sobolewski, P., Sołtysiak, A. *The orthogonal complement of $\mathcal{M}(a)$ in $\mathcal{H}(b)$*, Studia Math. 260 (2021), no. 3.