Reflection groups, Weyl chambers and Laplacians

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Let W be a finite reflection group associated with a root system R in R^d. Let C_+ denote a positive Weyl chamber distinguished by a choice of R_+, a set of positive roots. We investigate realizations in $L^2(C_+)$ of the Laplacian on C_+, subject to mixed Dirichlet-Neumann boundary conditions imposed on the facets of C_+. These conditions are determined by a homomorphism $\eta \in \mathrm{Hom}(W, \widehat{Z}_2)$, where $\widehat{Z}_2 = \{1, -1\}$ with multiplication. Thorough analysis of the corresponding η-heat kernels together with proof of their positivity on C_+ is also discussed.