1 Title: HECKE ALGEBRA ON HOMOGENEOUS TREES AND RELATIONS WITH HANKEL AND TOEPLITZ OPERATORS

2 Abstract:

On a homogeneous tree X_q of degree $q \geq 2$ (which means that every vertex has q edges), with the distance function $\texttt{dist}(x, y)$ defined on vertices, we consider an algebra of distance-dependent kernels $\chi_n(x, y) = 1$ if the distance between these vertices satisfies $\texttt{dist}(x, y) = n$ and $\chi_n(x, y) = 0$ otherwise, with $n = 0, 1, 2, \ldots$. These kernels form a commutative algebra $\mathbb{H}(X_q)$, called the Hecke algebra, which is generated by χ_1 and the main topic of the talk is to study when it is a maximal abelian subalgebra (MASA) in some bigger algebras. In a purely algebraic case, using some geometric argument, we show that this is the case in the algebra $\mathcal{F}(X_q)$ of kernels φ such that there exists $m \in \mathbb{N}$ for which $\varphi(x, y) = 0$ if $\texttt{dist}(x, y) > m$. In the case $q = 2$ the tree can be identified with integers $\mathbb{Z}$ and the Hecke algebra is considered as subalgebra of the algebra $\mathcal{Z}_1(\mathbb{Z})$ of operators acting simultaneously on $l^1(\mathbb{Z})$ and on $l^\infty(\mathbb{Z})$. In this case we show that the commutant of the Hecke subalgebra in $\mathcal{Z}_1(\mathbb{Z})$ is a direct sum of Hankel and Toeplitz operators. In the general case of $q \geq 2$ we consider the completion $\mathcal{F}_1(X_q)$ of $\mathcal{F}(X_q)$ with similar norm, which gives smaller algebra then $\mathcal{Z}_1(X_q)$ if $q = 2$. Then we show that the Hecke sublgebra is MASA in $\mathcal{F}_1(X_q)$.